#### Stat 201: Introduction to Statistics

Standard 26: Confidence Intervals – for Means

#### Means Sampling Distributions

Recall:

- The mean of the sampling distribution for a sample mean
  - $\mu_{\bar{x}} = the mean of all possible sample means = \mu_x = the population mean$
- The standard error, the standard deviation of all sample means, is:

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

#### **Confidence Intervals**

- Often, we do not know the population mean
- We use our sample means to make inference on the population parameter
- We MUST make sure that the data is obtained through randomization and that distribution of the data is approximately normal
  - Recall Central limit theorem:
    - For this we require n>30 or the population to be normal to begin with

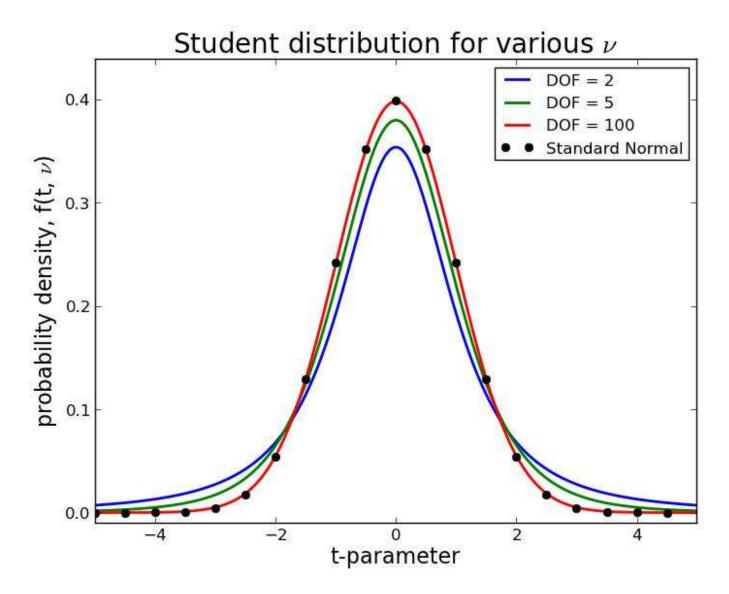
#### Confidence Intervals For the Population Mean

- When we talk about confidence intervals for the population mean we have two approaches
  - 1. When we know  $\sigma_{\chi}$  (we are rarely in this case)
    - Here we'll use the z-statistic
  - 2. When we don't know  $\sigma_x$ 
    - Here we'll use the t-statistic
    - T is very similar to Z
    - Degrees of freedom = sample size 1 = n-1

#### Properties of the t-distribution

- 1. The t-distribution is different for different degrees of freedom
- 2. The t-distribution is centered and symmetric at 0
- 3. The area under the curve is 1 and ½ on either side of 0
- 4. The probability approaches 0 as we move away from 0
- 5. The t-distribution has fatter tails than the standard normal
- 6. As the sample size increases t gets close to z

#### The t-distribution



#### Confidence Intervals When We Know $\sigma_{\chi}$

- $\bar{x}$  is our **point-estimate** for the population mean
  - Our 'best' guess for the true population , mean is our sample mean

#### Confidence Intervals When We Know $\sigma_{\chi}$

• We use our sample means to make inference on the population mean

$$\bar{x} \pm z_{1-\frac{\alpha}{2}} \left( \frac{\sigma_x}{\sqrt{n}} \right)$$

•  $\bar{x}$  is our **point-estimate** for the population mean

• 
$$z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_{\chi}}{\sqrt{n}}\right)$$
 is our margin of error

## Confidence Intervals: Margin of Error When We Know $\sigma_x$

- $z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_{\chi}}{\sqrt{n}}\right)$  is our margin of error
  - As n increases,  $\left(\frac{\sigma_x}{\sqrt{n}}\right)$  decreases, causing the margin of error to decrease causing the width of the confidence interval to narrow
  - As n decreases,  $\left(\frac{\sigma_x}{\sqrt{n}}\right)$  increases, causing the margin of error to increase causing the width of the confidence interval to widen

Confidence Intervals: Margin of Error When We Know $\sigma_x$ 

- $Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_{\chi}}{\sqrt{n}}\right)$  is our margin of error
  - As the confidence level decreases, z decreases causing the margin of error to decrease, causing the width of the confidence interval to narrow
  - As the confidence level increases, z increases
     causing the margin of error to increase, causing
     the width of the confidence interval to grow wider

Confidence Intervals Bounds When We Know  $\sigma_{\chi}$ Lower Bound =  $\bar{x} - z_{1-\frac{\alpha}{2}} \left(\frac{\sigma_{\chi}}{\sqrt{n}}\right)$ Upper Bound =  $\bar{x} + z_{1-\frac{\alpha}{2}} \left(\frac{\sigma_{\chi}}{\sqrt{n}}\right)$ 

We are --% confident that the true population mean,  $\mu_x$ , is between the **lower** and **upper** bound.

## Confidence Intervals When We Don't Know $\sigma_x$

• We use our sample means to make inference on the population mean

$$\bar{x} \pm t_{1-\frac{\alpha}{2},n-1} \left( \frac{S_{\chi}}{\sqrt{n}} \right)$$

•  $\bar{x}$  is our **point-estimate** for the population mean

• 
$$t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_x}{\sqrt{n}}\right)$$
 is our margin of error  
-  $s_x$  is the sample standard deviation

Confidence Intervals: Margin of Error When We Don't Know  $\sigma_x$ 

- $t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$  is our margin of error
  - As n increases, t decreases and  $\left(\frac{s_x}{\sqrt{n}}\right)$  decreases, causing the margin of error to decrease causing the width of the confidence interval to narrow
  - As n decreases, t increases and  $\left(\frac{s_x}{\sqrt{n}}\right)$  increases, causing the margin of error to increase causing the width of the confidence interval to widen

Confidence Intervals: Margin of Error When We Don't Know  $\sigma_x$ 

- $t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$  is our margin of error
  - As the confidence level decreases, t decreases causing the margin of error to decrease, causing the width of the confidence interval to narrow
  - As the confidence level increases, t increases
     causing the margin of error to increase, causing
     the width of the confidence interval to grow wider

Confidence Intervals Bounds When We Don't Know  $\sigma_{\chi}$ Lower Bound =  $\bar{x} - t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$ Upper Bound =  $\bar{x} + t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$ 

 We are --% confident that the true population mean, μ, is between the lower and upper bounds.

### Confidence Intervals When We Don't Know $\sigma_x$

- t is based on the t distribution which is a lot like the normal distribution but with fatter tails
  - You can find the correct t-value by finding the cross-hair of degrees of freedom, n-1, and the two tailed alpha
  - <u>http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf</u>

#### Finding t for Our Confidence Intervals

- Say we were trying to find the t-value for a 95% confidence with n=10
- This means  $\alpha = 1 .95 = .05$  and the degrees of freedom = 10 1 = 9

• 
$$t_{1-\frac{.05}{2},9} = 2.262$$

| cum. prob | t.50  | t.75<br>0.25 | t.80  | t.85<br>0.15 | t.90           | t.95  | t.975<br>0.025 | t.99  | t.995 | t.999  | t.9995                                  |
|-----------|-------|--------------|-------|--------------|----------------|-------|----------------|-------|-------|--------|---|
| two-tails | 1.00  | 0.50         | 0.40  | 0.30         | 0.20           | 0.10  | 0.05 B         | 0.02  | 0.01  | 0.002  | 0.001                                   |
| df        |       |              |       |              | and the second |       |                |       |       |        | 100000000000000000000000000000000000000 |
| 1         | 0.000 | 1.000        | 1.376 | 1.963        | 3.078          | 6.314 | 12.71          | 31.82 | 63.66 | 318.31 | 636.62                                  |
| 2         | 0.000 | 0.816        | 1.061 | 1.386        | 1.886          | 2.920 | 4.303          | 6.965 | 9.925 | 22.327 | 31.599                                  |
| 3         | 0.000 | 0.765        | 0.978 | 1.250        | 1.638          | 2.353 | 3.182          | 4.541 | 5.841 | 10.215 | 12.924                                  |
| 4         | 0.000 | 0.741        | 0.941 | 1.190        | 1.533          | 2.132 | 2.776          | 3.747 | 4.604 | 7.173  | 8.610                                   |
| 5         | 0.000 | 0.727        | 0.920 | 1.156        | 1.476          | 2.015 | 2.571          | 3.365 | 4.032 | 5.893  | 6.869                                   |
| 6         | 0.000 | 0.718        | 0.906 | 1.134        | 1.440          | 1.943 | 2.447          | 3.143 | 3.707 | 5.208  | 5.959                                   |
| 7         | 0.000 | 0.711        | 0.896 | 1.119        | 1.415          | 1.895 | 2.365          | 2.998 | 3.499 | 4.785  | 5.408                                   |
| 8         | 0.000 | 0.706        | 0.889 | 1.108        | 1.397          | 1.860 | 2.306          | 2.896 | 3.355 | 4.501  | 5.041                                   |
| A 9       | 0.000 | 0.703        | 0.883 | 1.100        | 1.383          | 1.833 | 2.262          | 2.821 | 3.250 | 4.297  | 4.781                                   |
| 10        | 0.000 | 0.700        | 0.879 | 1.093        | 1.372          | 1.812 | 2.228          | 2.764 | 3.169 | 4.144  | 4.587                                   |
|           |       |              |       |              |                |       |                |       |       |        |   |

### Zoom In

| cum. prob<br>one-tail | t.50<br>0.50 | t.75<br>0.25 | t.80<br>0.20 | t.85<br>0.15 | t.90<br>0.10   | t.95<br>0.05 | t .975<br>0.025 | t.99<br>0.01 | t.995<br>0.005 | t.999<br>0.001 | t.9995    |
|-----------------------|--------------|--------------|--------------|--------------|----------------|--------------|-----------------|--------------|----------------|----------------|-----------|
| two-tails             | 1.00         | 0.50         | 0.40         | 0.30         | 0.20           | 0.10         | 0.05            | 0.02         | 0.01           | 0.002          | 0.001     |
| df                    |              | 6.950 (July) | 1.5.20000000 |              | and the second |              | 10400-001       |              |                |                | NUMBER OF |
| 1                     | 0.000        | 1.000        | 1.376        | 1.963        | 3.078          | 6.314        | 12.71           | 31.82        | 63.66          | 318.31         | 636.62    |
| 2                     | 0.000        | 0.816        | 1.061        | 1.386        | 1.886          | 2.920        | 4.303           | 6.965        | 9.925          | 22.327         | 31.599    |
| 3                     | 0.000        | 0.765        | 0.978        | 1.250        | 1.638          | 2.353        | 3.182           | 4.541        | 5.841          | 10.215         | 12.924    |
| 4                     | 0.000        | 0.741        | 0.941        | 1.190        | 1.533          | 2.132        | 2.776           | 3.747        | 4.604          | 7.173          | 8.610     |
| 5                     | 0.000        | 0.727        | 0.920        | 1.156        | 1.476          | 2.015        | 2.571           | 3.365        | 4.032          | 5.893          | 6.869     |
| 6                     | 0.000        | 0.718        | 0.906        | 1.134        | 1.440          | 1.943        | 2.447           | 3.143        | 3.707          | 5.208          | 5.959     |
| 7                     | 0.000        | 0.711        | 0.896        | 1.119        | 1.415          | 1.895        | 2.365           | 2.998        | 3.499          | 4.785          | 5.408     |
| 8                     | 0.000        | 0.706        | 0.889        | 1.108        | 1.397          | 1.860        | 2.306           | 2.896        | 3.355          | 4.501          | 5.041     |
| A 9                   | 0.000        | 0.703        | 0.883        | 1.100        | 1.383          | 1.833        | 2.262           | 2.821        | 3.250          | 4.297          | 4.781     |
| 10                    | 0.000        | 0.700        | 0.879        | 1.093        | 1.372          | 1.812        | 2.228           | 2.764        | 3.169          | 4.144          | 4.587     |
|                       |              |              |              |              |                |              |                 |              |                |                |           |

- A is the degrees of freedom, n-1
- B is the significance level for confidence intervals we look for  $\alpha$  in the two-tail row
- C is the t-value associated with the provided degrees of freedom and significance level

#### Finding t for Our Confidence Intervals

- Say we were trying to find the t-value for a 99% confidence with n=9
- This means  $\alpha = 1 .99 = .01$  and the degrees of freedom = 9 1 = 8

• 
$$t_{1-\frac{.01}{2},8}=3.355$$

| 1.00          | 0.25   | 0.20  | 0.15  | 0.10<br>0.20   | 0.05<br>0.10   | 0.025<br>0.05  | 0.01<br>0.02  | t.995<br>0.005<br>0.01   | 0.001<br>B0.002   | 0.0005<br>0.001  |
|---------------|--|---|---|--|--|--|---|--|---|--|
| Sectores to 1 | SCROUPS:   | 15.02000 C  | N7 19725-   | CHARSON A  | 2016-28  | 0/#825303  | 4800260   | or same  | PROTECTION N  | passager;  |
| 0.000         | 1.000  | 1.376   | 1.963   | 3.078  | 6.314  | 12.71  | 31.82   | 63.66  | 318.31  | 636.62   |
| 0.000         | 0.816  | 1.061   | 1.386   | 1.886  | 2.920  | 4.303  | 6.965   | 9.925  | 22.327  | 31.599   |
| 0.000         | 0.765  | 0.978   | 1.250   | 1.638  | 2.353  | 3.182  | 4.541   | 5.841  | 10.215  | 12.924   |
| 0.000         | 0.741  | 0.941   | 1.190   | 1.533  | 2.132  | 2.776  | 3.747   | 4.604  | 7.173   | 8.610  |
| 0.000         | 0.727  | 0.920   | 1.156   | 1.476  | 2.015  | 2.571  | 3.365   | 4.032  | 5.893   | 6.869  |
| 0.000         | 0.718  | 0.906   | 1.134   | 1.440  | 1.943  | 2.447  | 3.143   | 3.707  | 5.208   | 5.959  |
| 0.000         | 0.711  | 0.896   | 1 1 1 9   | 1 415  | 1.895  | 2.365  | 2 998   | 3 499  | 4 785   | 5 408  |
| 0.000         | 0.706  | 0.889   | 1.108   | 1.397  | 1.860  | 2.306  | 2.896   | 3.355  | C 4.501   | 5.041  |
| 0.000 0.000   | 0.703<br>0.700   | 0.883<br>0.879  | 1.100<br>1.093  | 1.383<br>1.372   | 1.833<br>1.812   | 2.262<br>2.228   | 2.821<br>2.764  | 3.250<br>3.169   | 4.297<br>4.144  | 4.781<br>4.587   |
|               | 1.00<br>0.000<br>0.000<br>0.000<br>0.000<br>0.000<br>0.000<br>0.000<br>0.000 | 0.000         1.000           0.000         0.816           0.000         0.765           0.000         0.741           0.000         0.727           0.000         0.718           0.000         0.716           0.000         0.718           0.000         0.706           0.000         0.703 | 1.000.500.400.0001.0001.3760.0000.8161.0610.0000.7650.9780.0000.7410.9410.0000.7270.9200.0000.7180.9060.0000.7110.8960.0000.7060.8890.0000.7030.883 | 1.000.500.400.300.0001.0001.3761.9630.0000.8161.0611.3860.0000.7650.9781.2500.0000.7410.9411.1900.0000.7270.9201.1560.0000.7180.9061.1340.0000.7110.8961.1190.0000.7060.8891.1080.0000.7030.8831.100 | 1.000.500.400.300.200.0001.0001.3761.9633.0780.0000.8161.0611.3861.8860.0000.7650.9781.2501.6380.0000.7410.9411.1901.5330.0000.7270.9201.1561.4760.0000.7180.9061.1341.4400.0000.7110.89611191.4150.0000.7060.8891.1081.3970.0000.7030.8831.1001.383 | 1.000.500.400.300.200.100.0001.0001.3761.9633.0786.3140.0000.8161.0611.3861.8862.9200.0000.7650.9781.2501.6382.3530.0000.7410.9411.1901.5332.1320.0000.7270.9201.1561.4762.0150.0000.7180.9061.1341.4401.9430.0000.7110.8961.1191.4151.8950.0000.7060.8891.1081.3971.8600.0000.7030.8831.1001.3831.833 | 1.000.500.400.300.200.100.050.0001.0001.3761.9633.0786.31412.710.0000.8161.0611.3861.8862.9204.3030.0000.7650.9781.2501.6382.3533.1820.0000.7410.9411.1901.5332.1322.7760.0000.7270.9201.1561.4762.0152.5710.0000.7180.9061.1341.4401.9432.4470.0000.7110.8961.1191.4151.8952.3650.0000.7060.8891.1081.3971.8602.3060.0000.7030.8831.1001.3831.8332.262 | 1.000.500.400.300.200.100.050.020.0001.0001.3761.9633.0786.31412.7131.820.0000.8161.0611.3861.8862.9204.3036.9650.0000.7650.9781.2501.6382.3533.1824.5410.0000.7410.9411.1901.5332.1322.7763.7470.0000.7270.9201.1561.4762.0152.5713.3650.0000.7180.9061.1341.4401.9432.4473.1430.0000.7110.8961.1191.4151.8952.3652.9980.0000.7060.8891.1081.3971.8602.3062.8960.0000.7030.8831.1001.3831.8332.2622.821 | 1.000.500.400.300.200.100.050.020.010.0001.0001.3761.9633.0786.31412.7131.8263.660.0000.8161.0611.3861.8862.9204.3036.9659.9250.0000.7650.9781.2501.6382.3533.1824.5415.8410.0000.7410.9411.1901.5332.1322.7763.7474.6040.0000.7270.9201.1561.4762.0152.5713.3654.0320.0000.7180.9061.1341.4401.9432.4473.1433.7070.0000.7110.8961.1191.4151.8952.3652.9983.4990.0000.7060.8891.1081.3971.8602.3062.8963.3550.0000.7030.8831.1001.3831.8332.2622.8213.250 | 1.00         0.50         0.40         0.30         0.20         0.10         0.05         0.02         0.01         B0.002           0.000         1.000         1.376         1.963         3.078         6.314         12.71         31.82         63.66         318.31           0.000         0.816         1.061         1.386         1.886         2.920         4.303         6.965         9.925         22.327           0.000         0.765         0.978         1.250         1.638         2.353         3.182         4.541         5.841         10.215           0.000         0.741         0.941         1.190         1.533         2.132         2.776         3.747         4.604         7.173           0.000         0.718         0.906         1.134         1.440         1.943         2.447         3.143         3.707         5.208           0.000         0.711         0.896         1.119         1.415         1.895         2.365         2.998         3.499         4.785           0.000         0.706         0.889         1.108         1.397         1.860         2.306         2.896         3.355         C         4.501           0.000 |

|                                    |                      |                      |                      | Zo                   | om                               | In                   |                        |                      |                        |                          |                           |
|------------------------------------|----------------------|----------------------|----------------------|----------------------|----------------------------------|----------------------|------------------------|----------------------|------------------------|--------------------------|---------------------------|
| cum. prob<br>one-tail<br>two-tails | t.50<br>0.50<br>1.00 | t.75<br>0.25<br>0.50 | t.80<br>0.20<br>0.40 | t.85<br>0.15<br>0.30 | t <sub>.90</sub><br>0.10<br>0.20 | t.95<br>0.05<br>0.10 | t.975<br>0.025<br>0.05 | t.99<br>0.01<br>0.02 | t.995<br>0.005<br>0.01 | t.999<br>0.001<br>B0.002 | t.9995<br>0.0005<br>0.001 |
| df                                 | Contraction of       | A-240-24             | 12.0000000           |                      | and the second                   | 10000                | 1042104201             |                      |                        | -                        | States No.                |
| 1                                  | 0.000                | 1.000                | 1.376                | 1.963                | 3.078                            | 6.314                | 12.71                  | 31.82                | 63.66                  | 318.31                   | 636.62                    |
| 2                                  | 0.000                | 0.816                | 1.061                | 1.386                | 1.886                            | 2.920                | 4.303                  | 6.965                | 9.925                  | 22.327                   | 31.599                    |
| 3                                  | 0.000                | 0.765                | 0.978                | 1.250                | 1.638                            | 2.353                | 3.182                  | 4.541                | 5.841                  | 10.215                   | 12.924                    |
| 4                                  | 0.000                | 0.741                | 0.941                | 1.190                | 1.533                            | 2.132                | 2.776                  | 3.747                | 4.604                  | 7.173                    | 8.610                     |
| 5                                  | 0.000                | 0.727                | 0.920                | 1.156                | 1.476                            | 2.015                | 2.571                  | 3.365                | 4.032                  | 5.893                    | 6.869                     |
| 6                                  | 0.000                | 0.718                | 0.906                | 1.134                | 1.440                            | 1.943                | 2.447                  | 3.143                | 3.707                  | 5.208                    | 5.959                     |
| 7                                  | 0.000                | 0.711                | 0.896                | 1 1 1 9              | 1 415                            | 1.895                | 2 365                  | 2 998                | 3,499                  | 4 785                    | 5 408                     |
| A 8                                | 0.000                | 0.706                | 0.889                | 1.108                | 1.397                            | 1.860                | 2.306                  | 2.896                | 3.355                  | C 4.501                  | 5.041                     |
| 9<br>10                            | 0.000 0.000          | 0.703<br>0.700       | 0.883<br>0.879       | 1.100<br>1.093       | 1.383<br>1.372                   | 1.833<br>1.812       | 2.262<br>2.228         | 2.821<br>2.764       | 3.250<br>3.169         | 4.297<br>4.144           | 4.781<br>4.587            |

• A is the degrees of freedom, n-1

- B is the significance level for confidence intervals we look for  $\alpha$  in the two-tail row
- C is the t-value associated with the provided degrees of freedom and significance level

#### Finding t for Our Confidence Intervals

- Say we were trying to find the t-value for a 90% confidence with n=11
- This means  $\alpha = 1 .90 = .10$  and the degrees of freedom = 11 1 = 10

• 
$$t_{1-\frac{.10}{2},10} = 1.812$$

| cum. prob<br>one-tail<br>two-tails | t.50<br>0.50<br>1.00 | t.75<br>0.25<br>0.50 | t.80<br>0.20<br>0.40 | t.85<br>0.15<br>0.30 | t <sub>.90</sub><br>0.10<br>0.20 | t.95<br>0.05<br>0.10 | t.975<br>0.025<br>B 0.05 | t.99<br>0.01<br>0.02 | t.995<br>0.005<br>0.01 | t.999<br>0.001<br>0.002 | t.9995<br>0.0005<br>0.001 |
|------------------------------------|----------------------|----------------------|----------------------|----------------------|----------------------------------|----------------------|--------------------------|----------------------|------------------------|-------------------------|---------------------------|
| df                                 |                      | Alexandra -          | in control of        | and the second       | and the second                   |                      | 10425-041                | a potencial          | 000000000              |                         | Markey INC.               |
| 1                                  | 0.000                | 1.000                | 1.376                | 1.963                | 3.078                            | 6.314                | 12.71                    | 31.82                | 63.66                  | 318.31                  | 636.62                    |
| 2                                  | 0.000                | 0.816                | 1.061                | 1.386                | 1.886                            | 2.920                | 4.303                    | 6.965                | 9.925                  | 22.327                  | 31.599                    |
| 3                                  | 0.000                | 0.765                | 0.978                | 1.250                | 1.638                            | 2.353                | 3.182                    | 4.541                | 5.841                  | 10.215                  | 12.924                    |
| 4                                  | 0.000                | 0.741                | 0.941                | 1.190                | 1.533                            | 2.132                | 2.776                    | 3.747                | 4.604                  | 7.173                   | 8.610                     |
| 5                                  | 0.000                | 0.727                | 0.920                | 1.156                | 1.476                            | 2.015                | 2.571                    | 3.365                | 4.032                  | 5.893                   | 6.869                     |
| 6                                  | 0.000                | 0.718                | 0.906                | 1.134                | 1.440                            | 1.943                | 2.447                    | 3.143                | 3.707                  | 5.208                   | 5.959                     |
| 7                                  | 0.000                | 0.711                | 0.896                | 1.119                | 1.415                            | 1.895                | 2.365                    | 2.998                | 3.499                  | 4.785                   | 5.408                     |
| 8                                  | 0.000                | 0.706                | 0.889                | 1.108                | 1.397                            | 1.860                | 2.306                    | 2.896                | 3.355                  | 4.501                   | 5.041                     |
| 9                                  | 0.000                | 0 703                | 0.883                | 1,100                | 1.383                            | 1.833                | 2 262                    | 2 821                | 3 250                  | 4 297                   | 4 781                     |
| A 10                               | 0.000                | 0.700                | 0.879                | 1.093                | 1.372                            | 1.812                | C 2.228                  | 2.764                | 3.169                  | 4.144                   | 4.587                     |

|                                    |                      |                      |                      | Zo                   | om                               | In                   |                          |                      |                                    |                         |                           |
|------------------------------------|----------------------|----------------------|----------------------|----------------------|----------------------------------|----------------------|--------------------------|----------------------|------------------------------------|-------------------------|---------------------------|
| cum. prob<br>one-tail<br>two-tails | t.50<br>0.50<br>1.00 | t.75<br>0.25<br>0.50 | t.so<br>0.20<br>0.40 | t.ss<br>0.15<br>0.30 | t <sub>.90</sub><br>0.10<br>0.20 | t.95<br>0.05<br>0.10 | t.975<br>0.025<br>B 0.05 | t.99<br>0.01<br>0.02 | t <sub>.995</sub><br>0.005<br>0.01 | t.999<br>0.001<br>0.002 | t.9995<br>0.0005<br>0.001 |
| df                                 |                      | Contraction of the   | A CONTRACTOR OF A    | NO SERVICE           | and an orall                     | and the              | 0/812530                 | 12000760             | 000400000                          | SHORES NO.              | procession of             |
| 1                                  | 0.000                | 1.000                | 1.376                | 1.963                | 3.078                            | 6.314                | 12.71                    | 31.82                | 63.66                              | 318.31                  | 636.62                    |
| 2                                  | 0.000                | 0.816                | 1.061                | 1.386                | 1.886                            | 2.920                | 4.303                    | 6.965                | 9.925                              | 22.327                  | 31.599                    |
| 3                                  | 0.000                | 0.765                | 0.978                | 1.250                | 1.638                            | 2.353                | 3.182                    | 4.541                | 5.841                              | 10.215                  | 12.924                    |
| 4                                  | 0.000                | 0.741                | 0.941                | 1.190                | 1.533                            | 2.132                | 2.776                    | 3.747                | 4.604                              | 7.173                   | 8.610                     |
| 5                                  | 0.000                | 0.727                | 0.920                | 1.156                | 1.476                            | 2.015                | 2.571                    | 3.365                | 4.032                              | 5.893                   | 6.869                     |
| 6                                  | 0.000                | 0.718                | 0.906                | 1.134                | 1.440                            | 1.943                | 2.447                    | 3.143                | 3.707                              | 5.208                   | 5.959                     |
| 7                                  | 0.000                | 0.711                | 0.896                | 1.119                | 1.415                            | 1.895                | 2.365                    | 2.998                | 3.499                              | 4.785                   | 5.408                     |
| 8                                  | 0.000                | 0.706                | 0.889                | 1.108                | 1.397                            | 1.860                | 2.306                    | 2.896                | 3.355                              | 4.501                   | 5.041                     |
| 9                                  | 0.000                | 0 703                | 0.883                | 1,100                | 1.383                            | 1.833                | 2 262                    | 2 821                | 3 250                              | 4 297                   | 4 781                     |
| A 10                               | 0.000                | 0.700                | 0.879                | 1.093                | 1.372                            | 1.812                | C 2.228                  | 2.764                | 3.169                              | 4.144                   | 4.587                     |

- A is the degrees of freedom, n-1
- B is the significance level for confidence intervals we look for  $\alpha$  in the two-tail row
- C is the t-value associated with the provided degrees of freedom and significance level

## Confidence Interval Bounds When We Don't Know $\sigma_x$

$$\bar{x} \pm t_{1-\frac{\alpha}{2},n-1}\left(\frac{S_{\chi}}{\sqrt{n}}\right)$$

Lower Bound= 
$$\bar{x} - t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$$
  
Upper Bound=  $\bar{x} + t_{1-\frac{\alpha}{2},n-1}\left(\frac{s_{\chi}}{\sqrt{n}}\right)$ 

- Suppose a random sample of 81 students from the University of South Carolina was taken. Among the sampled students the sample mean number of times they inappropriately used the word like in a five minute conversation was 13 times with a sample standard deviation of 2.
- Our sample mean =  $\bar{x}$  = 13
- Our sample standard deviation =  $s_{\chi}$  = 2

- Among the sampled students the sample mean number of times they inappropriately used the word like in a five minute conversation was 13 times with a sample standard deviation of 2.
- Check Assumptions
  - n>30 so it is safe to assume the distribution of x
     is bell-shaped
  - The data is from a random sample

 95% Confidence Interval for population mean number of times a University of South Carolina student inappropriately says like in a five minute conversation:

$$\bar{x} \pm t_{1-\frac{.05}{2},80} \left(\frac{S_{x}}{\sqrt{n}}\right)$$
$$= 13 \pm (1.990) \left(\frac{2}{\sqrt{81}}\right)$$
$$(11.5578, 14.4422)$$

#### (11.5578, 14.4422)

 We are 95% confident that the true population mean number of times a University of South Carolina student inappropriately says like in a five minute conversation is between 11.5578 and 14.4422 times

- Suppose a random sample of 38 yearly average temperature measures in New Haven, CT. Among the sampled years the sample mean temperature was 51.0474 degrees
   Fahrenheit with a sample standard deviation of 1.3112.
- Our sample mean =  $\bar{x}$  = 51.0474
- Our sample standard deviation =  $s_{\chi}$  = 1.3112

- Suppose a random sample of 38 yearly average temperature measures in New Haven, CT. Among the sampled years the sample mean temperature was 51.0474 degrees Fahrenheit with a sample standard deviation of 1.3112.
- Check Assumptions
  - n>30 so it is safe to assume the distribution of x
     is bell-shaped
  - The data is from a random sample

 95% Confidence Interval for population the true population mean yearly average temperature reading in New Haven is:

$$\bar{x} \pm t_{1-\frac{.05}{2},38-1} \left(\frac{S_{x}}{\sqrt{n}}\right)$$
$$= 51.0474 \pm (2.021) \left(\frac{1.3112}{\sqrt{38}}\right)$$
$$(50.61752, 51.47728)$$

#### (50.61752, 51.47728)

 We are 95% confident that the true population mean yearly average temperature reading in New Haven is between 50.61752 and 51.47728 degrees Fahrenheit

- Confidence Intervals for means TI83/84
  - <u>https://www.youtube.com/watch?v=H3uU-Tx2Yq0</u>
- Raw Data
  - <u>https://www.youtube.com/watch?v=k2tV34JniHc</u>
  - <u>https://www.youtube.com/watch?v=uUXfr8pZAO0</u>

## Confidence Intervals for Means on

your TI Calculator

• When we know  $\sigma_{\chi}$ , with data

#### • <u>INPUT:</u>

- 1. Press STAT
- 2. Press  $\rightarrow$  to TESTS
- 3. Highlight '7: ZInterval'
- 4. Press ENTER

#### 5. <u>With Data</u>

- 1. Enter the population standard deviation next to ' $\sigma$ :'
- 2. You should have your data table entered in L1
  - If you forgot: Press STAT, Press ENTER with 'Edit' highlighted, Enter the data into the L1 col.
- 3. Next to 'List:' press 2<sup>nd</sup> then press 1
- 4. Set 'Frequency' to 1
- 5. Enter the desired Confidence Level next to 'C-Level:'
- 6. Highlight Calculate
- 7. Press ENTER

- When we know  $\sigma_{\chi}$ , with data
- <u>OUTPUT:</u>
  - (lower bound, upper bound) is our confidence interval
  - $-\bar{x}$  is the sample mean for the problem
  - $-s_{\chi}$  is the sample standard deviation for the problem
  - n is the sample size and should match the number you entered

- When we know  $\sigma_{\chi}$ , with stats
- <u>INPUT:</u>
  - 1. Press STAT
  - 2. Press  $\rightarrow$  to TESTS
  - 3. Highlight '7: ZInterval'
  - 4. Press ENTER

#### 5. With Stats

- 1. Enter the population standard deviation next to ' $\sigma$ :'
- 2. Put the sample mean next to ' $\bar{x}$ :'
- 3. Put the sample size next to 'n:'
- 4. Enter the desired Confidence Level next to 'C-Level:'
- 5. Highlight Calculate
- 6. Press ENTER

- When we know  $\sigma_{\chi}$ , with stats
- <u>OUTPUT:</u>
  - (lower bound, upper bound) is our confidence interval
  - $-\,\bar{x}$  is the sample mean for the problem and should match the number you entered
  - n is the sample size and should match the number you entered

## Confidence Intervals for Means on

your TI Calculator

- When we don't know  $\sigma_{\chi}$ , with data
- <u>INPUT:</u>
  - 1. Press STAT
  - 2. Press  $\rightarrow$  to TESTS
  - 3. Scroll down using  $\downarrow$  to highlight '8: TInterval'
  - 4. Press ENTER

#### <u>With Data</u>

- 1. You should have your data table entered in L1
  - If you forgot: Press STAT, Press ENTER with 'Edit' highlighted, Enter the data into the L1 col.
- 2. Next to 'List:' press 2<sup>nd</sup> then press 1
- 3. Set 'Frequency' to 1
- 4. Enter the desired Confidence Level next to 'C-Level:'
- 5. Highlight Calculate
- 6. Press ENTER

- When we don't know  $\sigma_{\chi}$ , with data
- <u>OUTPUT:</u>
  - (lower bound, upper bound) is our confidence interval
  - $-\bar{x}$  is the sample mean for the problem
  - $-s_{\chi}$  is the sample standard deviation for the problem
  - n is the sample size and should match the number you entered

- When we don't know  $\sigma_{\chi}$ , with stats
- <u>INPUT:</u>
  - 1. Press STAT
  - 2. Press  $\rightarrow$  to TESTS
  - 3. Scroll down using  $\downarrow$  to highlight '8: TInterval'
  - 4. Press ENTER

#### 5. With Stats

- 1. Put the sample mean next to ' $\bar{x}$ :'
- 2. Enter the sample standard deviation next to ' $s_x$ :'
- 3. Put the sample size next to 'n:'
- 4. Enter the desired Confidence Level next to 'C-Level:'
- 5. Highlight Calculate
- 6. Press ENTER

- When we don't know  $\sigma_{\chi}$ , with stats
- <u>OUTPUT:</u>
  - (lower bound, upper bound) is our confidence interval
  - $\bar{x}$  is the sample mean for the problem and should match the number you entered in stem 6b
  - $-s_{\chi}$  is the sample standard deviation for the problem
  - n is the sample size and should match the number you entered in step 6c above

# Confidence Intervals for Means unknown: When we don't know $\sigma_x$ • StatCrunch Commands w/ data

- Stat→T Stats→One Sample
   →with data (if you have the a list of data)→Choose the column→choose confidence interval→enter the significance level → Compute
- <u>StatCrunch Commands w/ summaries</u>

Stat→T Stats→One Sample
 → with summary (if you have the count) → enter
 the mean, standard deviation and sample size→
 choose confidence interval→enter the significance
 level → Compute

Confidence Intervals for Means unknown: When we know  $\sigma_x$ • StatCrunch Commands w/ data

Stat→Z Stats→One Sample
 →with data (if you have the a list of data)→Choose the column→choose confidence interval→enter the significance level → Compute

#### <u>StatCrunch Commands w/ summaries</u>

Stat→Z Stats→One Sample
 → with summary (if you have the count) → enter
 the mean, standard deviation and sample size→
 choose confidence interval→enter the significance
 level → Compute

#### Confidence Intervals known $\sigma_x$

| Assumptions   | Point<br>Estimate | Margin of Error                                | Margin of Error  |
|---|-------------------|--|--|
| <ol> <li>Random Sample</li> <li>n &gt; 30 OR the population is bell shaped</li> </ol> | $\overline{x}$    | $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$ | $\bar{x} \pm \frac{z\alpha}{2} \left(\frac{\sigma_x}{\sqrt{n}}\right)$ |

• We are --% confident that the true population mean lays on the confidence interval.

#### Confidence Intervals unknown $\sigma_x$

| Assumptions   | Point<br>Estimate | Margin of<br>Error                        | Margin of Error   |
|---|-------------------|---|---|
| <ol> <li>Random Sample</li> <li>n &gt; 30 OR the population is bell shaped</li> </ol> | $\overline{x}$    | $\sigma_{\bar{x}} = \frac{S_x}{\sqrt{n}}$ | $\bar{x} \pm t_{1-\frac{\alpha}{2},n-1} \left(\frac{S_{\chi}}{\sqrt{n}}\right)$ |

• We are --% confident that the true population mean lays on the confidence interval.